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Viscosity of Liquid Toluene at Temperatures from 25 to 150 °C and at Pressures up to 30 MPa

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New measurements are presented for the viscosity of liquid toluene at temperatures from 25 to 150 °C and at pressures up to 30 MPa. The measurements were obtained with an oscillating-disk viscometer and have an estimated accuracy of ±0.5%. A comparison with data reported by other researchers is included, and an equation for the viscosity of liquid toluene as a function of temperature and density is given. Toluene was chosen because of its usefulness as a primary reference standard substance for thermal conductivity of liquids and its potential use as a similar standard for viscosity.

Introduction

In this paper we present accurate measurements of the viscosity of liquid toluene as a function of both temperature and pressure. The measurements were obtained with an oscillating-disk viscometer. Our decision to measure the viscosity of liquid toluene was based on the following considerations. Liquid toluene has been recommended as a primary reference standard for the thermal conductivity of liquids (*1–3*), and the question arises whether liquid toluene would also be a suitable reference standard for the viscosity of liquids (*4*, *5*). Accordingly, the Subcommittee of IUPAC Commission I.2 on the Transport Properties of Fluids has recommended that the viscosity of liquid toluene be measured in different laboratories with a variety of experimental techniques.

Oscillating-disk viscometers have proven to be reliable instruments to measure the viscosity of gases and, more recently, of liquids (*6*, *7*). The reason is that considerable theoretical information is available concerning the working equations for this method (*8*, *9*). The theory of the working equations has been reconsidered by Kestin and Shankland (*10*, *11*) and by Nieuwoudt et al. (*7*, *12*) with the purpose of investigating how the instrument can be used for measuring the viscosity of liquids both on an absolute and on a relative basis. Since the viscosity of liquid toluene near room temperature has been measured by a number of researchers, it was considered useful to make additional measurements by the oscillating-disk method and so to provide reliable values for its use as a calibration substance for the viscosity of liquids. Furthermore, toluene is a frequently used solvent, and reliable information about its physical properties, including viscosity, is also of direct technical importance.

Experimental Method

The viscosity was measured with an oscillating-disk viscometer developed by Kestin and co-workers at Brown University (*13*, *14*) and subsequently transferred to the University of Maryland (*15*). The viscometer is shown in Figure 1. Nearly all the wetted parts are made of Hastelloy C276 for resistance to corrosion. The viscometer body consists of two large pieces. The upper piece 7 defines the cavity which is filled with the liquid to be investigated. The cavity volume is about 1.3 dm^3. The lower piece 3 carries a Bridgman window, 2, a filling port, 19, and a multijunction thermocouple probe, 16. The two pieces are held together by a stainless steel cap, 8, which threads onto the lower piece and bears on the upper piece via ten bolts, 12, and a pressure ring, 13. The two pieces of the viscometer are sealed against an internal pressure up to 30 MPa by means of a Viton O-ring, 17, or, for use at temperatures above 200 °C, by a metal C-ring, 20. The lower piece 3 rests on a self-centering roller bearing, 1. An oscillation of the disk is initiated by manually turning the viscometer through a small angle, waiting approximately one half-period, and then returning it to the original position.

The oscillating disk 5 with a radius of 33.972 mm, a height of 3.21 mm, and a mass of 102.1 g is suspended between two fixed horizontal plates, 4 and 6. The gap between the disk and each plate is 2.249 mm. The purpose of the presence of the fixed plates is to suppress any possible convective flow of the liquid near the disk. The oscillating disk is mounted to a torsion wire, 9, via a cylindrical chuck with a radius of 2.79 mm and a height of 26.33 mm. The stress-relieved torsion wire has a length of 256 mm and a diameter of 0.13 mm; it is made of a

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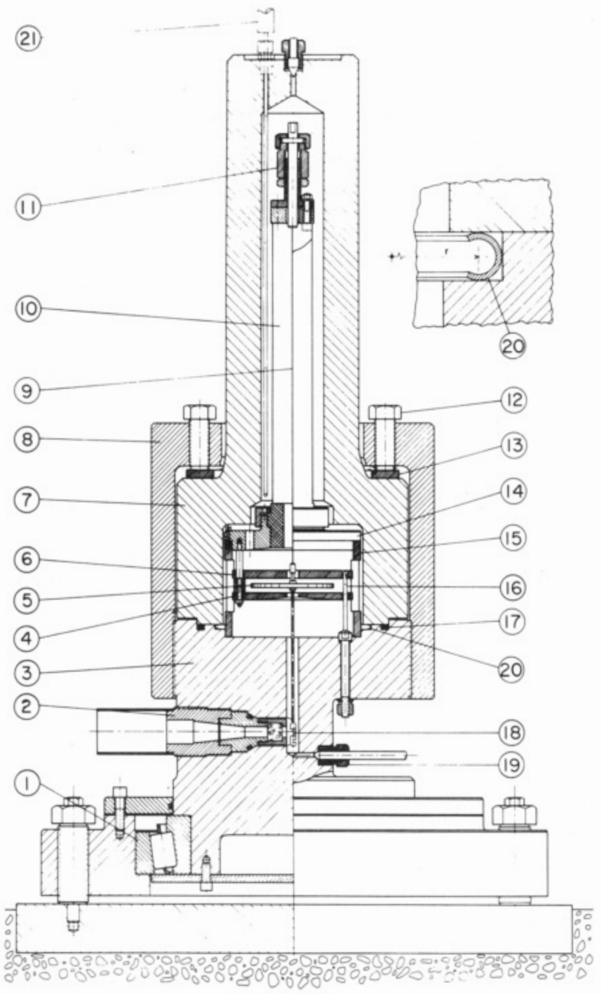

Figure 1. Oscillating-disk viscometer.

Table I. Characteristics of the Oscillating System

radius of disk R	33.972 mm
height of disk $2h$	3.210 mm
spacing between disk and plates b	2.249 mm
density of disk $\bar{\rho}$	8.7941 g cm^{-3}
moment of inertia I	59 093 g mm^2
length of torsion wire	256 mm
diameter of torsion wire	0.13 mm
period T_0 in vacuo at 25 °C	16.7894 s
damping rate Δ_0 in vacuo at 25 °C	3.4×10^{-5}

92% Pt–8% W alloy, since this material has a small internal damping and gives a high reproducibility of the rest position (*16*). The wire support and adjustment assembly 11 is carried by an alumina column, 10, which is open over most of its height. This alumina column is attached at its bottom to the suspension mounting plate 14 which is in turn attached to the cylindrical holder 15 carrying the fixed plates 4 and 6. The lengths and thermal-expansion coefficients of the column, the wire, and holder are such that the disk remains centered between the plates 4 and 6 over the working temperature range of the instrument (*7*). The column and holder are constructed in such a way that, with the upper piece 7 of the viscometer removed, the disk and wire are readily accessible. This permits checking and adjusting of the true running of the disk, the coincidence of the bearing axis with the suspension wire, and the disk's position between the two fixed plates. The characteristic properties of the oscillating system are summarized in Table I.

The viscometer is filled by allowing liquid to flow through the lower port 19 from an elevated reservoir by gravity. A port at the top of the viscometer allows air to escape. The surfaces of the holder and column have been designed so as to avoid trapping air bubbles above the rising liquid. The system is pressurized with the aid of a hand pump, and the pressure is measured with a Bourdon gauge with an accuracy of 0.1 MPa.

The viscometer is heated by an electric furnace which consists of an insulated resistance wire wrapped on an aluminum core in bifilar windings. The inside diameters of the core match the outside diameters of the viscometer top and the stainless-steel cap. The power to the two zones is separately controlled by feedback controllers responding to thermocouple sensors in the windings of the furnace. The two-zone system allows a small positive temperature gradient to be imposed as a further safeguard against convection. The viscometer is designed for operation at temperatures up to 300 °C. However, in measuring the viscosity of toluene, we did not go beyond 150 °C. The problem is that the elastic properties of the torsion wire begin to vary appreciably at temperatures beyond 200 °C, causing some loss of accuracy at the higher temperatures (*14*, *16*).

Temperatures inside the viscometer are measured with copper–constantan thermocouples. Probe 21 with thermocouple junctions spaced over 30 cm allows measurement of the temperature t_w of the suspension wire, upon which the period T_o and the damping rate Δ_o in vacuo depend. Probe 16 contains four junctions, spaced over 1.8 cm, which monitor the temperature distribution around the disk. The third junction from the bottom is at the level of the disk, and its reading is taken as the reference temperature t of the viscosity measurement. A digital voltmeter allows measurement of the temperature to be made with a precision of ±0.03 °C. The temperature measurements are accurate to ±0.1 °C near room temperature and to ±0.6 °C at 150 °C.

After an initial deflection the disk suspended in the liquid executes a damped harmonic motion, so that the angular displacement $\alpha(\tau)$ as a function of the time τ satisfies the equation (*7–12*)

$$\alpha(\tau) = \alpha_o \exp(-2\pi\Delta\tau/T) \sin(2\pi\tau/T) \tag{1}$$

where α_o is the amplitude, T the period, and Δ the damping rate of the oscillations. The period T and the damping rate Δ are measured optically. A mirror, 18, is mounted to the oscillating disk through a cylindrical stem with a radius of 1.17 mm and a height of 101.6 mm. Laser light reflected from the mirror sweeps past two detectors, one located near the rest position of the oscillating beam and the other at an arbitrary, but fixed, offset angle. The period T and the damping rate Δ are then obtained from the time intervals registered by the detectors as described by Kestin and Shankland (*17*).

The period T_o of the oscillating system in vacuo was deduced from the period T_g measured in argon gas with a procedure described by Nieuwoudt et al. (*12*). It is represented by

$$T_o/\text{s} = 16.7894 + 2.115 \times 10^{-3}(t_w/°\text{C} - 25) + 7.6 \times 10^{-7}(t_w/°\text{C} - 25)^2 \tag{2}$$

where t_w is the temperature of the torsion wire. The period T_o is strictly speaking not the period in vacuo, but the period in the absence of the liquid. The effect of an elevated pressure on the oscillating system is primarily through the changed properties of the liquid, but there is also a small effect on the oscillating system directly resulting in a small dependence of T_o on the pressure P, such that $T_o^{-1}(\partial T_o/\partial P)_t = -9.2 \times 10^{-6}$ MPa^{-1}. This pressure correction is not important in measuring the viscosity, but it becomes relevant when one tries to use the oscillating-disk method for measuring liquid densities as well (*15*). The damping rate Δ_o may be similarly extracted from measurements in a gas; it has been previously determined by Nott as (*18*)

$$\Delta_o = d_0 + d_1(t_w/°\text{C}) + d_2(t_w/°\text{C})^2 + d_3(t_w/°\text{C})^3 \tag{3}$$

with $d_0 = 3.668 \times 10^{-5}$, $d_1 = -1.146 \times 10^{-7}$, $d_2 = 5.633 \times$

10^{-11}, and $d_3 = 1.612 \times 10^{-11}$. Further details concerning the oscillating-disk instrument and its operation are contained in the Ph.D. thesis of Krall (*19*).

Working Equation

The damped harmonic motion described by eq 1 is governed by the equation

$$(s + \Delta_0)^2 + 1 + D(s) = 0 \qquad (4)$$

with

$$s = (-\Delta + i)T_0/T \qquad (5)$$

where $D(s)$ is the scaled Laplace transform of the torque exerted by the liquid on the disk, while T_0 and Δ_0 are the period and damping rate of the oscillating motion in vacuo (*7–12*). The torque function $D(s)$ is related to the viscosity η and the density ρ of the liquid through the boundary layer thickness, defined as

$$\delta = (\eta T_0/2\pi\rho)^{1/2} \qquad (6)$$

If the density ρ of the liquid is known, the viscosity η of the liquid is deduced from the observed damping rate Δ and period T through the imaginary part of eq 4:

$$2(\Delta\theta - \Delta_0)\theta = Im\, D(s) \qquad (7)$$

with

$$\theta = T_0/T \qquad (8)$$

An exact expression for the torque function $D(s)$ of the actual oscillating-disk assembly is not available. However, for a disk with radius R, height $2h$ and density $\bar{\rho}$, oscillating between two fixed horizontal plates located at equal distances b above and below the disk, a suitable approximate expression is (*12*, *15*)

$$D(s) = (\rho\delta/\bar{\rho}h)[As^{3/2} + Bs(\delta/R) + Cs^{1/2}(\delta/R)^2] \qquad (9)$$

with

$$A = \coth(bs^{1/2}/\delta) + 4h/R \qquad (10)$$

$$B = (16/3\pi)(4\pi/27^{1/2} - 1) + 6h/R \qquad (11)$$

$$C = 17/9 + 3h/2R \qquad (12)$$

Equation 10 and the second terms of eqs 11 and 12 represent the torque function of a disk between two fixed plates without edge effects (*20*), while the first terms of eqs 11 and 12 represent an expansion in terms of δ/R to account for edge effects in the approximation that the effect of the presence of the fixed plates on these edge effects can be neglected (*9*).

To determine $Im\, D(s)$, it is convenient to write (*10*)

$$s^{1/2} = x + iy \qquad (13)$$

with

$$x = \{(\theta/2)[(\Delta^2 + 1)^2 - \Delta]\}^{1/2} \qquad (14)$$

$$y = \theta/2x \qquad (15)$$

and

$$s^{3/2} = -H_2 + iH_1 \qquad (16)$$

with

$$H_1 = 3\theta x/2 - (\theta/2x)^3 \qquad (17)$$

$$H_2 = 3\theta^2/4x - x^3 \qquad (18)$$

and

$$\coth(bs^{1/2}/\delta) = K_2(\delta) - i K_1(\delta) \qquad (19)$$

with

$$K_1(\delta) = \frac{\sin(\theta b/\delta x)}{\cosh(2xb/\delta) - \cos(\theta b/\delta x)} \qquad (20)$$

$$K_2(\delta) = \frac{\sinh(2xb/\delta)}{\cosh(2xb/\delta) - \cos(\theta b/\delta x)} \qquad (21)$$

We then obtain from eq 5 the final form of the working equation

$$2(\Delta\theta - \Delta_0)\theta = g_1\rho\delta[H_1K_2 + H_2K_1 + g_2H_1 + g_3\theta\delta + g_4(\theta/x)\delta^2 + E(\delta)] \qquad (22)$$

where g_i (i = 1–4) are instrument constants defined by

$$g_1 = (\bar{\rho}h)^{-1} = \pi R^4/I \qquad (23)$$

$$g_2 = 4h/R \qquad (24)$$

$$g_3 = B/R \qquad (25)$$

$$g_4 = C/2R^2 \qquad (26)$$

In eq 22 we have introduced an unknown function $E(\delta)$ to account for any deviations due to the theoretical approximations introduced in the derivation of the working equation. The working equation (eq 22) reduces to the working equation adopted by Kestin and Shankland (*10*), if the first terms in eqs 11 and 12 are taken to be zero. Inclusion of these terms has the advantage that the working equation (eq 22) with $E(\delta) = 0$ can be used to measure the viscosity of liquids on an absolute basis without calibration. As demonstrated in the next section, such a procedure yields liquid viscosities with accuracies at the percentage level. To obtain a higher accuracy, we retain the function $E(\delta)$ and determine it by calibrating with a reference liquid. The calibration function $E(\delta)$ in the working equation (eq 22) is related to the complex calibration function $\epsilon(s,\delta)$ introduced by Nieuwoudt et al. (*12*) through $E(\delta) = (h/R)Im[s^{3/2}\epsilon(i,\delta)]$.

Calibration

The function $E(\delta)$ in the working equation (eq 22) was determined by calibrating with liquid water. For this purpose the viscometer was filled with distilled water degassed by vigorous boiling. The period and damping rate were measured at temperatures from 22 to 156 °C, with the temperature intervals chosen so as to obtain roughly equal intervals in the boundary layer thickness δ. All measurements were made at a slightly elevated pressure of 1.5 MPa so as to be well above the saturation pressure at all temperatures and to avoid the formation of vapor bubbles around the disk. The density ρ_{H_2O} and the viscosity η_{H_2O} of the water were calculated from the equations recommended by Kestin et al. (*21*).

The values thus obtained for the function $E(\delta)$ in eq 22 are shown in Figure 2 as a function of the boundary layer thickness. The measurements correspond to a range from δ = 1.60 mm at room temperature to δ = 0.72 mm at the highest temperature. The function $E(\delta)$ can be represented by a cubic polynomial of the form

$$E(\delta) = e_0 + e_1(\delta/\text{mm}) + e_2(\delta/\text{mm})^2 + e_3(\delta/\text{mm})^3 \qquad (27)$$

with $e_0 = -0.075\,725$, $e_1 = 0.229\,537$, $e_2 = -0.205\,340$, and $e_3 = 0.055\,056$. Equation 27 reproduces the observed calibration values of $E(\delta)$ with a standard deviation of 0.000 36 which corresponds to an uncertainty in the measured viscosity of only 0.06%.

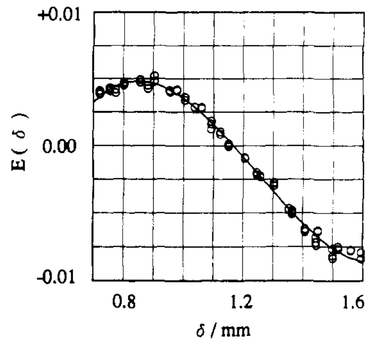

Figure 2. Calibration function $E(\delta)$ as a function of the boundary layer thickness δ.

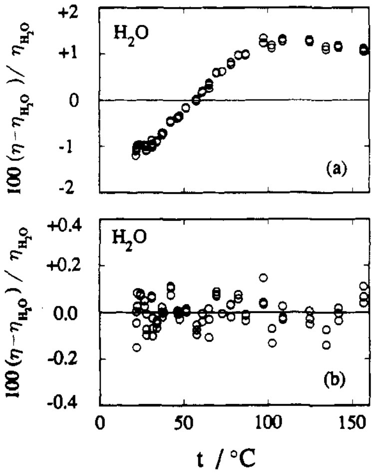

Figure 3. Comparison of the viscosity η measured for liquid water as a function of temperature with the reference viscosity η_{H_2O} from the literature, (a) when the viscometer is used in the absolute mode and (b) when the viscometer is used in the relative mode.

The actual accuracy of the measured viscosities will depend on the accuracy of the values of the viscosities as given by the equation of Watson et al. (*22*) used in this paper. At temperatures between 30 and 115 °C this equation differs from an alternate equation proposed by Kestin et al. (*23*) by less than −0.2%, but at 150 °C it differs by +0.4%. The estimated accuracy of ±0.4% at 150 °C causes some uncertainty in the curvature of $E(\delta)$ observed at the smaller boundary layer thickness values.

It is interesting to check to what extent our viscometer can yield viscosities of liquids on an absolute basis. When the viscometer is used in an absolute mode, the viscosity η is deduced from the measured damping rate and period through the working equation (eq 22) with $E(\delta) = 0$. The absolute viscosities thus obtained are compared with the reference viscosities in Figure 3a. We conclude that the instrument yields absolute viscosities of liquid water with an accuracy of about ±1%. When the viscometer is used in the relative mode, the viscosity η is deduced from the measured damping rate and period through the working equation (eq 22) with $E(\delta)$ represented by eq 27. The relative viscosities thus obtained are compared with the reference viscosities in Figure 3b. The relative viscosities are reproduced within ±0.2% which is within the estimated accuracy of the reference viscosities as discussed above.

Experimental Viscosities for Toluene

High-purity liquid chromatography (HPLC) grade toluene with a stated purity of 99.9% as supplied by Aldrich Chemical was used for the experiments. The viscosity was determined at nine temperatures from 24 to 151 °C and at seven pressures from atmospheric pressure up to 30 MPa. The measurements were obtained and analyzed in terms of temperatures based on the International Practical Temperature Scale of 1968 (IPTS-68). For the purpose of this paper all temperatures have been converted to the new International Temperature Scale of 1990 (ITS-90). For this purpose we represented the differences between the temperatures T_{68} and T_{90} on IPTS-68 and ITS-90 by a polynomial of the form

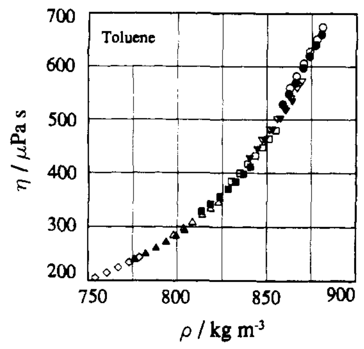

Figure 4. Measured viscosities of liquid toluene plotted as a function of density: ○, 24 °C; ●, 30 °C; ▽, 41 °C; ▼, 49 °C; □, 60 °C; ■, 77 °C; △, 98 °C; ▲, 121 °C; ◇, 152 °C.

$$(T_{68} - T_{90})/\mathrm{K} = a_1(T_{68}/\mathrm{K} - 273.16) + a_2(T_{68}/\mathrm{K} - 273.16)^2 + a_3(T_{68}/\mathrm{K} - 273.16)^3 \quad (28)$$

with $a_1 = 2.361\,26 \times 10^{-4}$, $a_2 = 5.504\,24 \times 10^{-7}$, and $a_3 = -3.619\,22 \times 10^{-9}$. Equation 28 is based on a table presented by McGlashan (*24*) and is valid in the range 270 K $\leq T \leq$ 440 K. (It should be noted that in this section the symbol T denotes temperatures in Kelvin and not period as in the previous sections.)

In order to deduce the viscosity from the observed damping rate and period through the working equation (eq 22), we need to know the density ρ of liquid toluene. For this purpose we adopted an equation proposed by Kashiwagi et al. (*25*):

$$\rho/(\mathrm{kg\ m^{-3}}) = \rho_0(T)\left\{1 - C \ln\left[\frac{D(T) + P/\mathrm{MPa}}{D(T) + 0.1}\right]\right\} \quad (29)$$

with $C = 0.091\,43$ and where the functions $\rho_0(T)$ and $D(T)$ are polynomial functions of the temperature T. This equation is based on experimental densities at temperatures from 0 to 100 °C and pressures up to 200 MPa. Here we have extrapolated it for use at temperatures up to 150 °C. After conversion of the temperatures to ITS-90, the functions $\rho_0(T)$ and $D(T)$ can be represented as

$$\rho_0(T) = 1103.15 - 0.68109(T/\mathrm{K}) - 0.4228 \times 10^{-3}(T/\mathrm{K})^2 \quad (29a)$$

and

$$D(T) = 440.60 - 1.6047(T/\mathrm{K}) + 1.540 \times 10^{-3}(T/\mathrm{K})^2 \quad (29b)$$

The experimental results obtained for the viscosity of toluene are presented in Table II; Figure 4, showing the viscosity as a function of density, gives an overview of the data set.

Errors in our measurements of the viscosity of liquid toluene may be ascribed to three sources: errors in the reference values of the viscosity of water used in the calibration of the viscometer, instrumental drifts or lack of long-term reproducibility, and systematic errors of such a nature as not to be removed by the calibration. The range of boundary layer thickness covered in the toluene measurements, 0.84 mm $\leq \delta \leq$ 1.45 mm, corresponds in water to temperatures in the range from 30 to 115 °C. The accuracy of the reference values of the viscosity of liquid water may be considered to be ±0.3% on this range (*22*, *23*). We tested the viscometer for long-term stability by performing a series of check measure-

Table II. Experimental Results for Toluene

t_{90}/°C	ρ/(kg m^{-3})	η/(μPa s)	t_{90}/°C	ρ/(kg m^{-3})	η/(μPa s)	t_{90}/°C	ρ/(kg m^{-3})	η/(μPa s)
				P = 0.1 MPa				
24.52	862.9	559.5	41.14	847.3	462.3	59.28	830.0	384.5
29.09	858.7	529.1	48.64	840.2	427.3	76.54	813.3	328.1
				P = 5.0 MPa				
24.56	866.7	581.8	48.65	844.6	444.9	98.22	797.9	286.1
29.35	862.3	548.5	59.40	834.6	399.9	121.02	776.0	240.5
41.17	851.5	480.6	76.59	818.4	342.2	151.28	746.4	194.4
				P = 10.0 MPa				
24.31	870.7	605.9	48.62	848.9	462.8	98.32	803.6	298.4
29.44	866.1	569.3	59.34	839.2	416.7	121.21	782.4	251.7
41.10	855.7	500.7	76.75	823.3	356.1	151.52	754.0	204.9
				P = 15.0 MPa				
24.39	874.2	628.3	48.63	853.0	480.8	98.83	808.5	309.7
28.64	870.5	597.3	59.37	843.5	432.5	121.51	788.3	262.6
41.10	859.6	520.4	76.92	828.0	369.7	152.39	760.5	214.4
				P = 20.0 MPa				
24.40	877.7	651.7	48.02	857.4	501.7	98.53	813.9	323.1
28.72	874.0	619.4	59.62	847.5	448.2	121.65	793.9	273.6
41.12	863.4	539.7	77.04	832.4	383.8	152.53	767.0	224.2
				P = 25.0 MPa				
24.46	881.0	674.7	48.71	860.6	517.0	98.79	818.5	335.2
28.99	877.2	640.5	59.80	851.3	464.2	121.72	799.3	284.7
41.23	866.9	558.6	77.16	836.7	398.0	152.68	773.1	234.0
				P = 30.0 MPa				
24.48	884.2	700.8	48.91	864.1	534.0	99.00	823.0	346.8
29.25	880.3	661.2	59.89	855.1	480.3	122.09	804.1	294.8
42.29	869.6	571.6	77.39	840.7	411.4	152.86	778.8	243.2

ments of the viscosity of water after the completion of the measurements of toluene. Over the range of temperatures from 23 to 153 °C the viscosities calculated from eq 22 with $E(\delta)$ given by eq 27 agreed with the reference viscosities to within ±0.3%. Apart from the inaccuracy of the working equation which is corrected by the calibration function $E(\delta)$, the largest source of systematic error in our measurement is in the determination of the temperature. Like all systematic errors, this one is partly removed by the relative mode of the measurements; however, because the condition of equal boundary layer thickness occurs in water and toluene at different temperatures and because water and toluene differ in their sensitivity to temperature, errors in the value assigned to the temperature have effects which are not entirely removed by a calibration function $E(\delta)$ that depends on the boundary layer thickness alone. From consideration of the accuracy of the temperature measurements and their effects on the viscosity measurements, we judge that this source of error contributes an uncertainty of ±0.2%. These three sources imply a total uncertainty of ±0.5%. Further details about the measurements can be found in the Ph.D. thesis of Krall (*19*).

In order to represent the experimental viscosities as a function of temperature and density, we have adopted an equation previously used by Nieuwoudt et al. for the viscosity of liquid isobutane (*26*):

$$\eta(\rho^*,T^*)/(\mu\text{Pa s}) = h_0 \exp\left(\frac{h_1}{T^*}+\frac{h_3}{T^{*3}}\right)\frac{\rho^*}{1-\rho^*(h_4+h_5T^*)} \qquad (30)$$

with

$$\rho^* = \rho/\rho_c \qquad T^* = T/T_c \qquad (31)$$

where ρ_c = 290.2 kg m^{-3} and T_c = 593.91 K are the critical density and critical temperature of toluene (*27*). The coefficients h_i in eq 30 are given in Table III. The deviations of the measured viscosities from those calculated with eq 30 are shown in Figure 5. The equation reproduces the experimental viscosities with a standard deviation of 0.2%.

Table III. Coefficients in Equation 30 for the Viscosity of Liquid Toluene

h_0	12.977182	h_4	0.271276
h_1	0.036136	h_5	0.048892
h_3	0.061266		

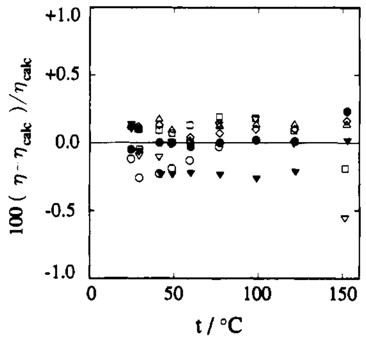

Figure 5. Relative deviations of the measured viscosity from the values η_{calc} calculated with the correlating equation (eq 30): ○, 0.1 MPa; ▽, 5.0 MPa; □, 10.0 MPa; △, 15.0 MPa; ◇, 20.0 MPa; ●, 25.0 MPa; ▼, 30.0 MPa.

Comparison with Other Data

As a parallel project the viscosity of liquid toluene at atmospheric pressure has been measured in our laboratory by Gonçalves et al. with a high-precision Ubbelohde viscometer at temperatures from 25 to 75 °C. The difference between our new viscosity measurements obtained with the oscillating-disk method and those obtained by Gonçalves et al. varies from +0.3% at 25 °C to −0.3% at 75 °C, which is within the accuracy of ±0.3% claimed by Gonçalves et al., but which reveals a somewhat different trend of the temperature dependence of η.

An extensive review of the viscosity of toluene at normal pressure has recently been presented by Laesecke et al. (*28*). An intercomparison of the various values reported for the vis-

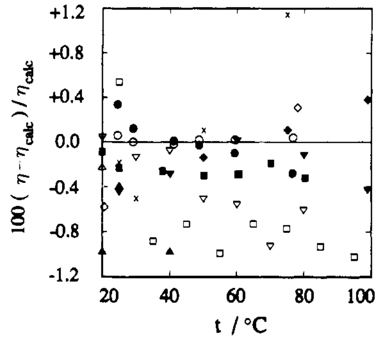

Figure 6. Viscosity η of liquid toluene at atmospheric pressure as a function of temperature, compared with the values η_{calc} from an Arrhenius equation (eq 32) recommended by Gonçalves et al. (*5*) ○, ref 5; ●, this work; ▽, ref 28; ▼, ref 32; □, ref 33; ■, ref 31; △, ref 4; ▲ ref 30; ◇ ref 29; ● ref 35; ×, ref 34.

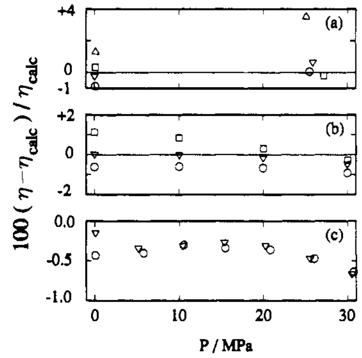

Figure 7. Comparison between the viscosity of liquid toluene at elevated pressures as reported in the literature with the values η_{calc} calculated from our correlating equation (eq 30): (a) falling-body technique (*35*); ○, 25 °C; ▽, 50 °C; □, 75 °C; △, 100 °C; (b) vibrating-crystal technique (*34*); ○, 25 °C; ▽, 50 °C; □, 75 °C; (c) vibrating-wire technique (*36*); ○, 30 °C; ▽, 50 °C.

cosity of toluene at normal pressure (*4*, *5*, *28–36*) is presented in Figure 6; in this figure the data are shown as differences from an Arrhenius equation:

$$\eta_{calc} = A \exp(B/T) \qquad (32)$$

with A = 16.09 μPa s and B = 1055.4 K as recommended by Gonçalves et al. (*5*). The measurements reported in refs 4, 5, and 28–33 were obtained with capillary-flow viscometers. All but one of these are measurements relative to a reference liquid, usually water. Heydweiller's result (*29*), an early absolute determination of the viscosity of water and of several other liquids, is the exception. A small correction has been applied to the measurements of Heydweiller (*29*) and of Geist and Cannon (*30*) so as to refer them to the current liquid-viscosity benchmark, namely, 1002 μPa s for the viscosity of water at 20 °C and atmospheric pressure (*3*, *37*). Whenever possible, comparison with eq 32 has been made by converting the measured kinematic viscosities to dynamic viscosities with eq 29 for the density ρ. The highest accuracy, ±0.2%, has been claimed by Hammond et al. (*31*) and Bauer and Meerlender (*4*), but their results consistently fall below those of Gonçalves et al. (*5*) by amounts sometimes exceeding 0.3%, but within 0.5%.

Three determinations of the viscosity of toluene at elevated pressures have been reported in the literature (*34–36*) with which our measurements can be compared; they are shown as deviations from our correlating equation (eq 30) in Figure 7. Kashiwagi and Makita measured the viscosity with a vibrating piezoelectric crystal; they did not report their original experimental data, but gave a formula for the pressure dependence of the viscosity on several isotherms from 25 to 75 °C which agrees with our results within the claimed accuracy of 2%. Dymond et al. (*35*) used a falling-body viscometer. At atmospheric pressure their results agree with those of Gonçalves et al. to within ±0.5% as shown in Figure 6; at elevated pressures they agree with our measurements within the claimed accuracy of ±4%. Assael et al. (*36*) used a vibrating wire viscometer. Their data show small but systematic negative deviations from our results. However, except at the highest pressure of 30 MPa, these deviations are within the accuracy of ±0.5% we claim for the correlating equation (eq 30).

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Glossary

b	distance between disk and plates
E	calibration function
I	moment of inertia
P	pressure
R	radius of disk
s	Laplace-transform variable
t	temperature, °C
T	period or temperature, K
T_0	period in vacuo
T_c	critical temperature
T^*	reduced temperature ($=T/T_c$)
α	angular displacement
δ	boundary layer thickness
Δ	damping rate
Δ_0	damping rate in vacuo
θ	inverse reduced period ($=T_0/T$)
η	viscosity
ρ	density of liquid
ρ_c	critical density
ρ^*	reduced density ($=\rho/\rho_c$)
$\bar{\rho}$	density of disk

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